

Impact of losses and detuning on quantum vacuum emission

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A system in the ultrastrong coupling regime is predicted to emit photons if the intensity of the light-matter coupling is modulated in time. This phenomenon, that presents strong similarities with other forms of quantum vacuum emission like the dynamical Casimir effect and the Hawkins radiation, has been theoretically studied in various strongly coupled systems but to date eluded experimental verifications. In this paper we investigate the impact of losses and detuning on the intensity of the emitted radiation. We will prove that losses have a limited effect and that not only quasi-resonance is not required, but a red-detuned photonic mode can actually enhance the emission. Those results prove that strong coupling and quasi-resonance between light and matter excitations are not required to observe quantum vacuum emission upon modulation of light-matter coupling, opening novel venues to observe this fascinating but elusive effect.

I. INTRODUCTION

When the resonant coupling of an optically active transition with light is larger than its linewidth, it becomes possible to spectroscopically resolve the resonant splitting due to the interaction and the system is said to be in the strong light-matter coupling regime. In this regime the interaction between light and matter can not be described in terms of emission and absorption of photons, but it is necessary to consider instead the dressed light-matter excitations of the coupled system [1, 2]. If the coupling becomes instead comparable with the bare frequencies of the excitations, the system enters a different regime, that has been named ultrastrong coupling regime. Such a regime, described [3] and achieved [4] for the first time using intersubband polaritons, has since been studied both theoretically and experimentally in a variety of different systems [5–20]. The large interest in this novel regime has been fuelled by its rich phenomenology, including quantum phase transitions [21–24], modification of energy transport [25, 26] and light emission properties [27–29], and the possibility to use it to influence chemical and thermodynamic processes [30–33].

One of the first predictions linked with the ultrastrong coupling regime has been the possibility to emit photons out of the quantum vacuum when the strength of the interaction is non-adiabatically modulated in time, in a process that presents strong analogies with the dynamical Casimir effect [34–40] and with the Hawking radiation [41–43]. Notwithstanding a widespread interest [44–58], the possibility to obtain quantum vacuum emission through the modulation of an ultrastrongly coupled vacuum has until now eluded experimental observation due to the very challenging nature of the required experimental setup.

Interest in ultrastrong coupling has historically emerged from the study of strongly coupled systems, and its achievement is usually demonstrated measuring the coupling strength by fitting the resonant splitting of the coupled resonances. Any system in which ultrastrong coupling has been demonstrated was thus *a fortiori* also in the strong coupling regime and close enough to reso-

nance to identify the spectral anti-crossing. Still the two regimes depend on different figures of merit, and are thus *a priori* independent.

In this paper we will prove that neither strong coupling, nor quasi-resonance are necessary to achieve quantum vacuum emission out of a perturbed ultrastrongly coupled vacuum. Ultrastrong coupling, being an intrinsic property of the system, independent to a certain extent from its coupling with the external world, is more robust than strong coupling and this result thus opens novel opportunities to observe quantum vacuum radiation by exploiting systems with very large couplings but in which, for intrinsic or technological reasons, losses or the difficulty to realise resonant photonic resonators have impeached the observation of strong coupling. Prime examples could be graphene single and bilayers in which, notwithstanding different theoretical calculations predicting very large dipoles [18, 19], strong coupling has not been achieved yet, or hybrid quantum systems that, while recently highlighted as ideal platforms for some quantum vacuum emission scheme [44], have until now resisted efforts to reach the strong coupling regime being plagued by excessive losses [59, 60].

II. THEORY OF QUANTUM VACUUM EMISSION

We will start by reviewing a thought-experiment, originally from the seminal paper from Ciuti, Bastard, and Carusotto [3], and sketched in Fig. 1, that will help us to both gain a physical understanding of the reason a modulated ultrastrongly coupled vacuum emits light and also to identify the most relevant observable to estimate the emission intensity. Our first task will be to calculate perturbatively the ground state $|G\rangle$ of a photonic system coupled to an optically active transition. From standard perturbation theory the first order perturbative ground state will be of the form

$$|G\rangle \simeq |0\rangle + \sum_j \frac{\langle j| H_{\text{int}} |0\rangle}{E_j} |j\rangle, \quad (1)$$

where $|0\rangle$ is the ground state of the uncoupled light-matter system, $|j\rangle$ is an arbitrary excited state having bare energy E_j above the vacuum state, and H_{int} is the light-matter interaction Hamiltonian. Such a Hamiltonian contains both resonant terms, describing absorption and emission processes (annihilation of a photon and creation of a matter excitation and *vice versa*), and anti-resonant terms, that instead annihilate or create pairs of excitations [3]. Of those the last one, creating a pair of excitations, is the only one that contributes to Eq. (1), with a coefficient proportional to the ratio between the light-matter coupling strength and the sum of the bare energies of the light and matter excitations. That is to say, in the ultrastrong coupling regime, in which this ratio is non-negligible, we expect the coupled ground state to contain a finite population of virtual excitations. We now consider a system in which the interaction between light and matter can be modulated in time and it is non-adiabatically switched off at $t = \tau$. The ground state of the system will thus be $|G\rangle$ for $t < \tau$ and $|0\rangle$ for $t > \tau$. By definition of non-adiabaticity at $t = \tau^+$, just after the switch-off, the system will still be in the state $|G\rangle$, that it is now an excited state containing a population of free photons that can be detected. If we define $\hat{a}_{\mathbf{k}}$ to be the annihilation operator of a free photon mode indexed by the wavevector \mathbf{k} , the population of emitted photons in such a mode will thus be given by

$$N_k = \langle G | \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} | G \rangle. \quad (2)$$

This is of course an upper boundary on the emission efficiency, reached in the case of instantaneous switch-off, but it remains a reliable estimate while the switch-off happens on a timescale shorter than the optical period of the emitted radiation [52].

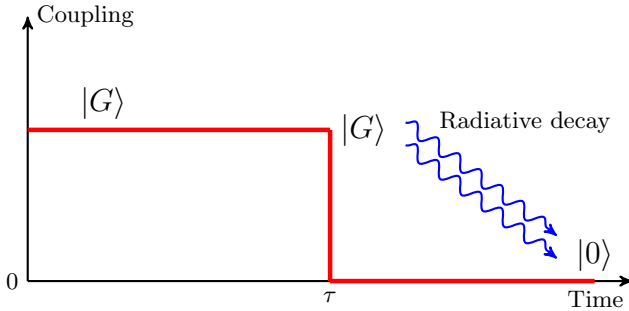


FIG. 1: The coupling between light and matter is non-adiabatically switched-off at $t = \tau$. At $t = \tau^+$ the system is still in the coupled ground state $|G\rangle$, which is in general different from the ground state of the uncoupled system $|0\rangle$. Being an excited state, it will eventually relax to the true ground state radiating its photonic population in the process.

Our aim will be to calculate the quantity in Eq. (2) in the most general linear dielectric system, for arbitrary detuning and dissipation. For sake of readability we will keep in the main body of the paper only the calculations

needed to follow the discussion, moving the more technical ones to the appendices, to which the interested reader will be referred for the details. Due to the regime we are interested in, with all the parameters *a priori* of the same order, a perturbative approach would be unreliable. We will thus perform a non-perturbative calculation based on the theory developed by Huttner and Barnett [61], who themselves extended the method originally due to Hopfield [62] to the case of a dispersive-dissipative dielectric. For sake of clarity we will consider an homogeneous dielectric medium, although the extension to the inhomogeneous case does not present any fundamental issue [63]. As explained in more details in Appendix A, we can perform a derivation very similar to the one of Ref. [61], starting from an Hamiltonian describing the electromagnetic field coupled to an optically active transition, itself coupled to a reservoir of harmonic oscillators acting as a bath in which energy can be dissipated. Introducing annihilation operators $\hat{C}(\mathbf{k}, \omega)$ for excitations of wavevector \mathbf{k} and frequency ω , obeying bosonic commutation relations

$$[\hat{C}(\mathbf{k}, \omega), \hat{C}^\dagger(\mathbf{k}', \omega')] = \delta(\mathbf{k} - \mathbf{k}')\delta(\omega - \omega'), \quad (3)$$

and using a method originally due to Fano [64], such an Hamiltonian can be put in the diagonal form

$$H = \int d^3k \int_0^\infty d\omega \hbar \omega \hat{C}^\dagger(\mathbf{k}, \omega) \hat{C}(\mathbf{k}, \omega). \quad (4)$$

The linear transformation used to diagonalize the system can then be inverted, allowing to express the photonic operators as linear combinations of the $\hat{C}(\mathbf{k}, \omega)$ as

$$\hat{a}(\mathbf{k}) = \int_0^\infty d\omega \left[\tilde{\alpha}_0^*(k, \omega) \hat{C}(\mathbf{k}, \omega) - \tilde{\beta}_0(k, \omega) \hat{C}^\dagger(-\mathbf{k}, \omega) \right], \quad (5)$$

with

$$\tilde{\alpha}_0(k, \omega) = \sqrt{\frac{\omega_c^2}{kc}} \left(\frac{\omega + kc}{2} \right) \frac{\zeta(\omega)}{\epsilon^*(\omega)\omega^2 - k^2c^2}, \quad (6)$$

$$\tilde{\beta}_0(k, \omega) = \sqrt{\frac{\omega_c^2}{kc}} \left(\frac{\omega - kc}{2} \right) \frac{\zeta(\omega)}{\epsilon^*(\omega)\omega^2 - k^2c^2},$$

where ω_c quantifies the coupling, the complex dielectric function is

$$\epsilon(\omega) = 1 + \frac{\omega_c^2}{2\omega} \int_{-\infty}^\infty d\omega' \frac{|\zeta(\omega')|^2}{\omega'(\omega' - \omega - i0^+)}, \quad (7)$$

and the functional form of $\zeta(\omega)$ can be found in Appendix A. By definition of ground state

$$\hat{C}(\mathbf{k}, \omega) |G\rangle = 0, \quad (8)$$

and we can calculate the number of emitted photons in the mode \mathbf{k} by inserting Eq. (5) into Eq. (2), and exploiting the commutation relation in Eq. (3), leading to

$$\begin{aligned} N_k &= \int_0^\infty d\omega |\tilde{\beta}_0(k, \omega)|^2 \\ &= \int_0^\infty d\omega \frac{(\omega - kc)^2}{2\pi kc} \frac{\text{Im}[\epsilon(\omega)] \omega^2}{|\epsilon(\omega)\omega^2 - k^2c^2|^2}, \end{aligned} \quad (9)$$

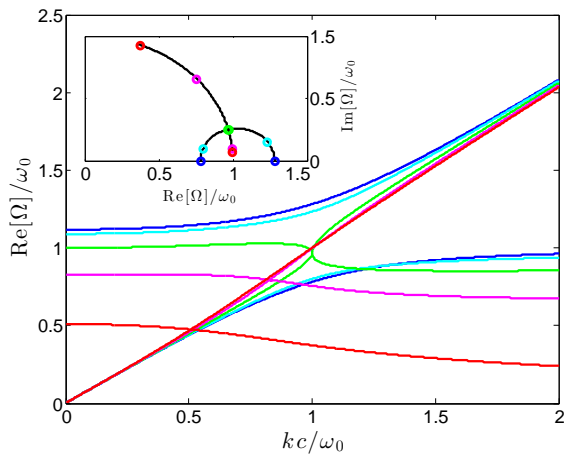


FIG. 2: Dispersion of the two polaritonic branches from the Lorentz model in Eq. (13), for $\omega_c = 0.5\omega_0$ and $\gamma = 0$ (blue), $0.5\omega_0$ (cyan), ω_0 (green), $1.5\omega_0$ (magenta), and $2\omega_0$ (red). Clearly visible is the transition between the strong coupling regime presenting and anticrossing (blue and cyan lines) and the weak one in which the polaritonic modes cross (magenta and red lines), with the green line at the edge between the two. In the inset we plot the trajectories that the eigenfrequencies draw in the complex plane, at resonance $kc = \omega_0$, while varying γ . Coloured circles mark the values of γ used in the main image.

where Im denotes the imaginary part, and we used Eq. (7) and the Sokhotski-Plemelj theorem to write

$$\text{Im}[\epsilon(\omega)] = \frac{\omega_c^2 \pi |\zeta(\omega)|^2}{2\omega^2}. \quad (10)$$

As detailed in Appendix C, the expression in Eq. (9) can be calculated through an integral in the complex plane in terms of the complex eigenfrequencies Ω_j , solutions of the dispersion equation in the denominator of the integral in Eq. (9)

$$|\epsilon(\omega)\omega^2 - k^2 c^2| = 0, \quad (11)$$

and the complex group velocities $\frac{d\Omega_j}{dk}$ as

$$N_k = \sum_j \text{Im} \left[\frac{\Omega_j^2 - k^2 c^2}{4\pi k^2 c^3} \frac{d\Omega_j}{dk} (i\pi - 2 \log(\Omega_j)) \right] - \frac{1}{2}, \quad (12)$$

where the sum in Eq. (12) is limited to eigenfrequencies in the first quadrant of the complex plane. In order to verify the correctness of our procedure, in Appendix D we compare the dissipationless limit of Eq. (12) to the result obtained using from scratch the Hopfield theory valid for nondissipative systems, showing that the two results coincide.

III. DISSIPATIVE LORENTZ MODEL

In order to study the physical content of Eq. (12) we apply it to a medium described by a dissipative Lorentz

dielectric function

$$\epsilon_L(\omega) = 1 + \frac{\omega_c^2}{\omega_0^2 - \omega^2 - i\gamma\omega}, \quad (13)$$

that is a medium containing a single, dispersionless optically active resonance of frequency ω_0 and linewidth γ . It is well known that the spectrum of a medium described by Eq. (13) consists of two polaritonic branches, crossing or anticrossing accordingly to whether the ratio $\frac{\gamma}{\omega_c}$ puts the system in the weak or in the strong coupling regime, as shown in Fig. 2. Such a model, with the appropriate choice of parameters, can be used to describe, at least qualitatively, all linear dielectric condensed matter systems in which strong and ultrastrong coupling have been achieved to date. In Appendix B we show that Eq. (13) is consistent with the form of the dielectric function in Eq. (7), and we can thus consistently apply it to Eq. (12). The poles of the dielectric function in Eq. (13),

$$\Omega_0 = \frac{-i\gamma \pm \sqrt{4\omega_0^2 - \gamma^2}}{2}, \quad (14)$$

that correspond to the complex frequencies of the uncoupled matter excitation, have a real component only for

$$\gamma < \gamma_{\max} = 2\omega_0, \quad (15)$$

thus fixing an upper bound for the physical value of the dissipation in such a model.

In Fig. 3 we plot the number of photons N_k emitted in the resonant mode $kc = \omega_0$ as a function of the coupling strength ω_c , for different values of γ ranging from 0 (blue line) to γ_{\max} (red line), obtained using the dielectric function in Eq. (13) to find the solutions of Eq. (11) and then plugging them into Eq. (12). We recognize a quadratic dependency of the emission over the coupling coefficient ω_c , as expected from the general perturbative argument exposed above. Remarkably we can notice that, while N_k does decrease with dissipation, even in the case of an overdamped oscillator it is only diminished of roughly 25% compared with the nondissipative case. As the coupling ω_c is varied from 0 to ω_0 we expect the system, at least for the small and intermediate values of γ , to transition from the weak to the strong coupling regime. Still no discontinuity is observed in N_k showing that strong coupling has no direct effect on quantum vacuum emission. This can be confirmed from the inset of Fig. 3 where we plot the trajectory of the two complex polaritonic eigenfrequencies in the complex plane for $\gamma = \omega_0$, as ω_c is varied, identifying with different symbols specific values marked in the main image. We can clearly see a transition from the weak to the strong coupling regime as the two eigenfrequencies transition from having different loss rates but similar frequencies to the opposite case.

Having clarified the role of dissipation in the quantum vacuum emission we now turn to study the role of the detuning between the optically active transition and

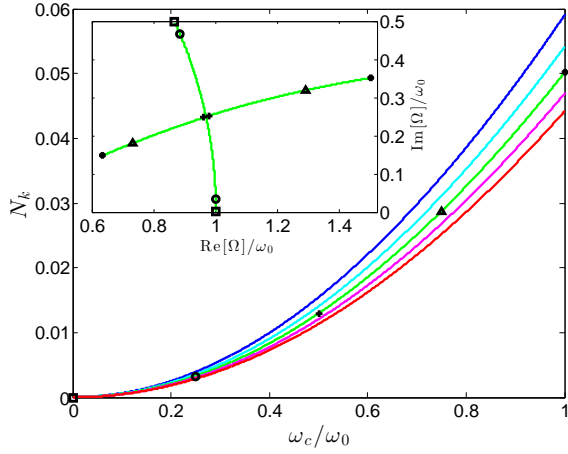


FIG. 3: Number of photons in the resonant mode $kc = \omega_0$ as a function of the coupling for $\gamma = 0$ (blue), $0.5\omega_0$ (cyan), ω_0 (green), $1.5\omega_0$ (magenta), and $2\omega_0$ (red), that is the maximal physical value for the model we are considering. We see that going from the undamped to the overdamped regime, the emission intensity only diminishes of around the 25%. In the inset we plot the trajectories that the eigenfrequencies draw in the complex plane, for $kc = \omega_0$ and $\gamma = \omega_0$, while varying ω_c . The black symbols in the main figure and the inset correspond to the same values of ω_c . The intensity of the emission presents no visible discontinuity passing from the weak to the strong coupling regime.

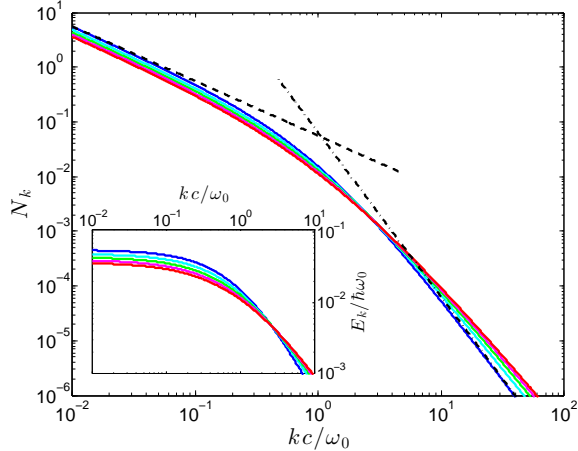


FIG. 4: Number of photons emitted as a function of the photonic wavevector for $\omega_c = 0.5\omega_0$ and $\gamma = 0$ (blue), $0.5\omega_0$ (cyan), ω_0 (green), $1.5\omega_0$ (magenta), and $2\omega_0$ (red). The black dashed and dash-dotted lines are the small and large k expansions from Eq. (16). In the inset is plotted instead the energy of the emitted radiation $E_k = \hbar kc N_k$.

the photonic mode. In Fig. 4 we plot N_k as a function of $\frac{kc}{\omega_0}$ over four orders of magnitude for a coupling $\omega_c = 0.5\omega_0$ and values of the dissipation covering all the possible range between 0 and γ_{\max} . We verify again that dissipation does not have any qualitative impact, and also its quantitative effect is negligible for a plot over multiple orders of magnitude of the wavevector. More

important, we do not see any sign of resonant enhancement of the emission. This can be understood again from our perturbative argument based on Eq. (1), from which it is clear that the mixing of the vacuum state with states containing photonic excitations is due to the nonresonant terms of the Hamiltonian, and thus no resonance condition should be expected. The emission intensity seems to have a dependency in k^{-1} for red-detuned photons and k^{-3} for blue-detuned ones. To verify this we can perform a perturbative development in inverse power of k (now justified as we are interested in extremal values of k) either from of the dissipationless version of Eq. (12) or using directly the Hopfield theory in Appendix D. Doing so we find in the two cases

$$N_k^{k \rightarrow 0} = \frac{\omega_c^2}{4kc\sqrt{\omega_0^2 + \omega_c^2}}, \quad (16)$$

$$N_k^{k \rightarrow \infty} = \frac{\omega_0\omega_c^2}{4k^3c^3}.$$

Those asymptotic behaviours are plotted as black lines in Fig. 4, where it is clear they fit well the overall trend. In the inset of Fig. 4 we plot instead the energy $E_k = \hbar kc N_k$ emitted per mode, showing that it is also a monotonously decreasing function of the photonic wavevector k . Using Eq. (16) we can estimate its upper value

$$E_{\max} = \frac{\hbar\omega_c^2}{4\sqrt{\omega_0^2 + \omega_c^2}}. \quad (17)$$

This result shows that light-matter quasi-resonance, while necessary to observe the strong-coupling and measure the coupling strength from the width of the spectral anticrossing, is not needed to observe quantum vacuum radiation, and it could even be deleterious. A strongly red-detuned photonic resonance seems in fact to have a number of desiderata: it allows to exploit higher-frequency strong dipolar transitions, like those found in organic materials, it allows to increase both the emitted energy and photon number, and it also potentially allows to reduce the requirements on the modulation frequency by targeting lower-frequency excitations. Notice that out-of-resonance ultrastrongly coupled systems have been investigated [67, 68] and realised [66] using circuit architectures, but the theory here developed for linear dielectric systems can not be directly applied to those superconducting, intrinsically non-linear systems.

IV. CONCLUSIONS AND PERSPECTIVES

In this paper we proved that the emission of light out of a perturbed quantum vacuum does not require neither strong coupling nor quasi-resonance. Those results, questioning what have been until now rather generally held assumptions, open novel paths toward a final observation of this fascinating effect: either exploiting systems characterised by very large dipoles but in which, for different

reasons, strong coupling has not been observed yet, or engineering new, more flexible devices, not bound by the requirement of quasi-resonance.

V. ACKNOWLEDGMENTS

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Appendix A: Diagonalization of a dispersive-dissipative dielectric

In their seminal work [61] Huttner and Barnett extended the microscopic quantum theory of light-matter interaction developed by Hopfield [62] to the case of homogeneous but lossy dielectrics. They accomplished this by quantising the electromagnetic field coupled to a dispersionless matter excitation, itself coupled to a continuum reservoir of harmonic oscillators acting as a bath in which energy could be dissipated. The full Hamiltonian of such a system can be written as

$$\hat{H} = \hat{H}_{\text{em}} + \hat{H}_{\text{mat}} + \hat{H}_{\text{int}} + \hat{H}_{\mathbf{A}^2}, \quad (\text{A1})$$

where

$$\hat{H}_{\text{em}} = \int d^3k \, \hbar k c \hat{a}^\dagger(\mathbf{k}) \hat{a}(\mathbf{k}), \quad (\text{A2})$$

describes the free electromagnetic field,

$$\begin{aligned} \hat{H}_{\text{mat}} = \int d^3k \, \Big\{ & \hbar \tilde{\omega}_0 \hat{b}^\dagger(\mathbf{k}) \hat{b}(\mathbf{k}) + \\ & \int_0^\infty d\omega \, \hbar \omega \hat{b}_\omega^\dagger(\mathbf{k}) \hat{b}_\omega(\mathbf{k}) + \int_0^\infty d\omega \, \frac{\hbar V(\omega)}{2} \\ & \left[\hat{b}^\dagger(-\mathbf{k}) + \hat{b}(\mathbf{k}) \right] \left[\hat{b}_\omega^\dagger(\mathbf{k}) + \hat{b}_\omega(-\mathbf{k}) \right] \Big\}, \end{aligned} \quad (\text{A3})$$

models the matter excitation and the bath,

$$\begin{aligned} \hat{H}_{\text{int}} = i \int d^3k \, \frac{\hbar \omega_c}{2} \sqrt{\frac{\tilde{\omega}_0}{k c}} \\ \left[\hat{a}^\dagger(-\mathbf{k}) + \hat{a}(\mathbf{k}) \right] \left[\hat{b}^\dagger(\mathbf{k}) - \hat{b}(-\mathbf{k}) \right], \end{aligned} \quad (\text{A4})$$

is the dipolar interaction between light and matter, and

$$\begin{aligned} \hat{H}_{\mathbf{A}^2} = \int d^3k \, \frac{\hbar \omega_c^2}{4 k c} \\ \left[\hat{a}^\dagger(-\mathbf{k}) + \hat{a}(\mathbf{k}) \right] \left[\hat{a}^\dagger(\mathbf{k}) + \hat{a}(-\mathbf{k}) \right], \end{aligned} \quad (\text{A5})$$

comes from the diamagnetic \mathbf{A}^2 part of the the minimal-coupling Hamiltonian [27, 28, 65]. In Eqs. (A2)-(A5) $a(\mathbf{k})$, $b(\mathbf{k})$, and $b_\omega(\mathbf{k})$, are bosonic annihilation operators respectively for a photon, a matter excitation, and an excitation of the bath with frequency ω , all indexed by

the wavevector \mathbf{k} , ω_0 is the frequency of the optically active transition, $V(\omega)$ models its coupling to the bath, ω_c quantifies the intensity of the light-matter coupling, and the renormalised frequency $\tilde{\omega}_0$ is linked to the bare one by the formula

$$\tilde{\omega}_0^2 = \omega_0^2 + \int_0^\infty \frac{|V(\omega)|^2 \tilde{\omega}_0}{\omega} d\omega. \quad (\text{A6})$$

Huttner and Barnett make the choice to remove $\hat{H}_{\mathbf{A}^2}$ by effectively performing a Bogoliubov rotation in the space of the $\hat{a}(\mathbf{k})$, thus putting $\hat{H}_{\text{em}} + \hat{H}_{\mathbf{A}^2}$ in diagonal form in terms of the Bogoliubov-rotated operators $\hat{\tilde{a}}(\mathbf{k})$

$$\hat{\tilde{H}}_{\text{em}} = \int d^3k \, \hbar \tilde{k} c \hat{\tilde{a}}^\dagger(\mathbf{k}) \hat{\tilde{a}}(\mathbf{k}), \quad (\text{A7})$$

with $\tilde{k} c = \sqrt{k^2 c^2 + \omega_c^2}$. Unluckily this is not acceptable for us, because it implies that the very definition of the the bare photonic operators depends upon the strength of the coupling, and as such it is not possible to properly define an equivalent of Eq. (2), because the photon operators with and without the coupling are defined in different ways. Ignoring this problem leads to a number of inconsistencies, most notably the total energy of the emitted quantum vacuum radiation per mode diverges for $k \rightarrow 0$. In this Appendix we will thus sketch a derivation that, while following the one of Ref. [61], keeps $\hat{H}_{\mathbf{A}^2}$ as part of the interaction.

The Hamiltonian in Eq. (A1) can be diagonalized *a la* Fano [64] in two steps. First \hat{H}_{mat} is put into diagonal form

$$\hat{H}_{\text{mat}} = \int d^3k \int_0^\infty d\omega \, \hbar \omega \hat{B}^\dagger(\mathbf{k}, \omega) \hat{B}(\mathbf{k}, \omega), \quad (\text{A8})$$

where the $\hat{B}(\mathbf{k}, \omega)$ operators, describing the continuously broadened optically active matter resonance, obey bosonic commutation relations

$$\left[\hat{B}(\mathbf{k}, \omega), \hat{B}^\dagger(\mathbf{k}', \omega') \right] = \delta(\mathbf{k} - \mathbf{k}') \delta(\omega - \omega'). \quad (\text{A9})$$

They can be expressed as linear combinations of the different uncoupled matter operators as

$$\begin{aligned} \hat{B}(\mathbf{k}, \omega) = \alpha_0(\omega) \hat{b}(\mathbf{k}) + \beta_0(\omega) \hat{b}^\dagger(-\mathbf{k}) \\ + \int_0^\infty d\omega' \left[\alpha_1(\omega, \omega') \hat{b}_{\omega'}(\mathbf{k}) + \beta_1(\omega, \omega') \hat{b}_{\omega'}^\dagger(-\mathbf{k}) \right], \end{aligned} \quad (\text{A10})$$

where the coefficients can be written as

$$\alpha_0(\omega) = \left(\frac{\omega + \tilde{\omega}_0}{2} \right) \frac{V(\omega)}{\omega^2 - \tilde{\omega}_0^2 z(\omega)}, \quad (\text{A11})$$

$$\beta_0(\omega) = \left(\frac{\omega - \tilde{\omega}_0}{2} \right) \frac{V(\omega)}{\omega^2 - \tilde{\omega}_0^2 z(\omega)},$$

$$\begin{aligned} \alpha_1(\omega, \omega') = \delta(\omega - \omega') \\ + \left(\frac{\tilde{\omega}_0}{2} \right) \left(\frac{V^*(\omega')}{\omega - \omega' - i0^+} \right) \frac{V(\omega)}{\omega^2 - \tilde{\omega}_0^2 z(\omega)}, \\ \beta_1(\omega, \omega') = \left(\frac{\tilde{\omega}_0}{2} \right) \left(\frac{V(\omega')}{\omega + \omega'} \right) \frac{V(\omega)}{\omega^2 - \tilde{\omega}_0^2 z(\omega)}, \end{aligned} \quad (\text{A12})$$

and

$$z(\omega) = 1 - \frac{1}{2\tilde{\omega}_0} \left[\int_{-\infty}^{\infty} d\omega' \frac{|V(\omega)|^2}{\omega' - \omega + i0^+} \right]. \quad (\text{A13})$$

Introducing the quantity

$$\zeta(\omega) = i\sqrt{\tilde{\omega}_0} [\alpha_0(\omega) + \beta_0(\omega)] = i \frac{\sqrt{\tilde{\omega}_0} \omega V(\omega)}{\omega^2 - \tilde{\omega}_0^2 z(\omega)}, \quad (\text{A14})$$

the interaction part of the Hamiltonian, now describing the interaction of the photonic modes with the broadened transition, takes the form

$$\hat{H}_{\text{int}} = \int d^3k \frac{\hbar\omega_c}{2\sqrt{kc}} \int_0^\infty d\omega \quad (\text{A15})$$

$$\left\{ \zeta(\omega) \hat{B}^\dagger(\mathbf{k}, \omega) [\hat{a}^\dagger(-\mathbf{k}) + \hat{a}(\mathbf{k})] + \text{H. c.} \right\}.$$

Analogously to what done before, the novel Hamiltonian can now be put into diagonal form

$$H = \int d^3k \int_0^\infty d\omega \hbar\omega \hat{C}^\dagger(\mathbf{k}, \omega) \hat{C}(\mathbf{k}, \omega), \quad (\text{A16})$$

through the polaritonic operators

$$\begin{aligned} \hat{C}(\mathbf{k}, \omega) &= \tilde{\alpha}_0(\omega) \hat{a}(\mathbf{k}) + \tilde{\beta}_0(\omega) \hat{a}^\dagger(-\mathbf{k}) \\ &+ \int_0^\infty d\omega' \left[\tilde{\alpha}_1(\omega, \omega') \hat{B}(\mathbf{k}, \omega') + \tilde{\beta}_1(\omega, \omega') \hat{B}^\dagger(-\mathbf{k}, \omega') \right], \end{aligned} \quad (\text{A17})$$

with the coefficients

$$\begin{aligned} \tilde{\alpha}_0(k, \omega) &= \sqrt{\frac{\omega_c^2}{kc}} \left(\frac{\omega + kc}{2} \right) \frac{\zeta(\omega)}{\epsilon^*(\omega)\omega^2 - k^2c^2}, \quad (\text{A18}) \\ \tilde{\beta}_0(k, \omega) &= \sqrt{\frac{\omega_c^2}{kc}} \left(\frac{\omega - kc}{2} \right) \frac{\zeta(\omega)}{\epsilon^*(\omega)\omega^2 - k^2c^2}, \\ \tilde{\alpha}_1(k, \omega, \omega') &= \delta(\omega - \omega') \\ &+ \frac{\omega_c^2}{2} \frac{\zeta^*(\omega')}{\omega - \omega' - i0^+} \frac{\zeta(\omega)}{\epsilon^*(\omega)\omega^2 - k^2c^2}, \quad (\text{A19}) \\ \tilde{\beta}_1(k, \omega, \omega') &= \frac{\omega_c^2}{2} \frac{\zeta^*(\omega')}{\omega - \omega' - i0^+} \frac{\zeta(\omega)}{\epsilon^*(\omega)\omega^2 - k^2c^2}, \end{aligned}$$

where the complex dielectric function is

$$\epsilon(\omega) = 1 + \frac{\omega_c^2}{2\omega} \int_{-\infty}^{\infty} d\omega' \frac{|\zeta(\omega')|^2}{\omega'(\omega' - \omega - i0^+)}. \quad (\text{A20})$$

For comparison the first coefficient obtained in Ref. [61] has instead the form

$$\tilde{\alpha}_0^{\text{HB}}(k, \omega) = \sqrt{\frac{\omega_c^2}{\tilde{k}c}} \left(\frac{\omega + \tilde{k}c}{2} \right) \frac{\zeta(\omega)}{\epsilon^*(\omega)\omega^2 - k^2c^2}, \quad (\text{A21})$$

with the others following a similar scheme.

Appendix B: Lorentz dielectric function

Here we will prove that the Lorentz dielectric function in Eq. (13) used through the paper is consistent with the general form in Eq. (7). We will do so by explicitly showing that the form in Eq. (13) is recovered by evaluating Eq. (7) with a coupling of the form

$$|V(\omega)|^2 = \frac{\omega\tilde{\omega}_0}{q + \omega_M}, \quad (\text{B1})$$

where ω_M is a cutoff frequency that we will eventually send to infinity and q a positive constant frequency. We will limit ourselves to consider the imaginary part of the dielectric function

$$\text{Im}[\epsilon(\omega)] = \frac{\omega_c^2 \tilde{\omega}_0^2 \text{Im}[z(\omega)]}{(\omega^2 - \tilde{\omega}_0^2 \text{Re}[z(\omega)])^2 + (\tilde{\omega}_0^2 \text{Im}[z(\omega)])^2}, \quad (\text{B2})$$

as the real part will be fixed by Kramers-Kronig relations. Plugging Eq. (B1) into Eq. (A6), and introducing the cutoff in the integration we obtain

$$\tilde{\omega}_0^2 = \omega_0^2 + \int_0^{\omega_M} \frac{|V(\omega)|^2 \tilde{\omega}_0}{\omega} d\omega = \omega_0^2 \frac{q + \omega_M}{q}. \quad (\text{B3})$$

From Eq. (A13), using the Sokhotski-Plemelj theorem

$$\begin{aligned} \text{Im}[z(\omega)] &= \frac{\pi}{2} \frac{|V(\omega)|^2}{\tilde{\omega}_0} = \frac{\pi}{2} \frac{\omega}{q + \omega_M}, \quad (\text{B4}) \\ \text{Re}[z(\omega)] &= 1 - \frac{1}{2\tilde{\omega}_0} \mathcal{P} \int_{-\omega_M}^{\omega_M} d\omega' \frac{|V(\omega')|^2}{\omega' - \omega} \\ &= 1 - \frac{\omega_M}{q + \omega_M} + \frac{\omega \log\left(1 - \frac{2\omega}{\omega + \omega_M}\right)}{2(q + \omega_M)}, \end{aligned}$$

where the \mathcal{P} indicates the principal part of the integral. Sending the cutoff to infinity we thus have

$$\begin{aligned} \lim_{\omega_M \rightarrow \infty} \tilde{\omega}_0^2 \text{Im}[z(\omega)] &= \frac{\pi}{2} \frac{\omega_0^2}{q} \omega, \quad (\text{B5}) \\ \lim_{\omega_M \rightarrow \infty} \tilde{\omega}_0^2 \text{Re}[z(\omega)] &= \omega_0^2. \end{aligned}$$

Using Eq. (B5) into Eq. (B2) we finally get

$$\text{Im}[\epsilon(\omega)] = \frac{\omega_c^2 \omega_0^2 \frac{\pi}{q} \frac{\omega}{2}}{(\omega^2 - \omega_0^2)^2 + (\omega_0^2 \frac{\omega}{q} \frac{\pi}{2})^2}, \quad (\text{B6})$$

that is the imaginary part of the Lorentz dielectric function

$$\epsilon_L(\omega) = 1 + \frac{\omega_c^2}{\omega_0^2 - \omega^2 - i\gamma\omega}, \quad (\text{B7})$$

upon the identification

$$\gamma = \frac{\pi\omega_0^2}{2q}. \quad (\text{B8})$$

Appendix C: Calculation of the complex integral

In order to calculate the integral in Eq. (9) we start by noticing, following Ref. [61], that the dielectric function calculated at a complex frequency Ω satisfies the relation

$$\epsilon(\Omega) = \epsilon^*(-\Omega^*), \quad (\text{C1})$$

and thus if Ω_j is a solution of the dispersion relation in Eq. (11), so are $-\Omega_j$, Ω_j^* , and $-\Omega_j^*$. Integrating over a keyhole contour in the complex plane, and developing the burdensome algebra paying attention to chose the principal values of Ω_j to have the brach cut on the positive real axis, we arrive at

$$\begin{aligned} N_k &= \int_0^\infty d\omega \frac{(\omega - kc)^2}{2\pi kc} \frac{\text{Im}[\epsilon(\omega)] \omega^2}{|\epsilon(\omega)\omega^2 - k^2c^2|^2} \\ &= \sum_j \left\{ \text{Im} \left[\frac{\Omega_j^2 - k^2c^2}{4\pi k^2c^3} \frac{d\Omega_j}{dk} (i\pi - 2\log(\Omega_j)) \right] \right. \\ &\quad \left. - \text{Re} \left[\frac{\Omega_j}{2kc^2} \frac{d\Omega_j}{dk} \right] \right\}, \end{aligned} \quad (\text{C2})$$

where the sum is only over the solutions in the first quadrant. At this point we can use the sum rule proved in Ref. [61]

$$\sum_j \text{Re} \left[\frac{\Omega_j}{k} \frac{d\Omega_j}{dk} \right] = c^2, \quad (\text{C3})$$

to obtain the result in Eq. (12).

Appendix D: Emission in the lossless case by Hopfield theory

In order to verify the correctness of our calculations, it is useful to compare the results obtained through the dissipative theory in the case of vanishing losses with those obtained using the standard nondissipative theory due to Hopfield [62]. In order to do this we start from the equivalent of Eq. (A1) neglecting the bath

$$\begin{aligned} \hat{H}_{\text{Hop}} &= \int d^3k \left\{ \hbar kc \hat{a}^\dagger(\mathbf{k}) \hat{a}(\mathbf{k}) + \hbar \omega_0 \hat{b}^\dagger(\mathbf{k}) \hat{b}(\mathbf{k}) \right. \\ &\quad \left. + \frac{\hbar \omega_c}{2} \sqrt{\frac{\omega_0}{kc}} [\hat{a}^\dagger(-\mathbf{k}) + \hat{a}^\dagger(\mathbf{k})] [\hat{b}(\mathbf{k}) + \hat{b}^\dagger(-\mathbf{k})] + \frac{\hbar \omega_c^2}{4kc} [\hat{a}^\dagger(-\mathbf{k}) + \hat{a}(\mathbf{k})] [\hat{a}^\dagger(\mathbf{k}) + \hat{a}(-\mathbf{k})] \right\}. \end{aligned} \quad (\text{D1})$$

The Hamiltonian in Eq. (D1) can be put in the diagonal form

$$\hat{H}_{\text{Hop}} = \int d^3k \sum_{j=\pm} \hbar \omega_{j,k} \hat{p}_j^\dagger(\mathbf{k}) \hat{p}_j(\mathbf{k}), \quad (\text{D2})$$

through the introduction of the polaritonic operators

$$\hat{p}_\pm(\mathbf{k}) = w_{\pm,k} \hat{a}(\mathbf{k}) + x_{\pm,k} \hat{b}(\mathbf{k}) + y_{\pm,k} \hat{a}^\dagger(-\mathbf{k}) + z_{\pm,k} \hat{b}^\dagger(-\mathbf{k}), \quad (\text{D3})$$

where

$$\omega_{\pm,k} = \sqrt{\frac{\omega_0^2 + \omega_c^2 + k^2c^2 \pm \sqrt{(\omega_0^2 + \omega_c^2 + k^2c^2)^2 - 4k^2c^2\omega_0^2}}{2}}, \quad (\text{D4})$$

and

$$\begin{pmatrix} w_{\pm,k} \\ x_{\pm,k} \\ y_{\pm,k} \\ z_{\pm,k} \end{pmatrix} = \left\{ \frac{\omega_{\pm,k}}{\omega_0} \left[\left(1 - \frac{\omega_{\pm,k}^2}{\omega_0^2} \right)^2 + \frac{\omega_c^2}{\omega_0^2} \right] \right\}^{-\frac{1}{2}} \begin{pmatrix} \left[1 - \frac{\omega_{\pm,k}^2}{\omega_0^2} \right] \frac{\omega_{\pm,k} + kc}{2\omega_0} \sqrt{\frac{\omega_0}{kc}} \\ -\frac{\omega_c}{2\omega_0} \left(1 + \frac{\omega_{\pm,k}}{\omega_0} \right) \\ - \left[1 - \frac{\omega_{\pm,k}^2}{\omega_0^2} \right] \frac{\omega_{\pm,k} - kc}{2\omega_0} \sqrt{\frac{\omega_0}{kc}} \\ -\frac{\omega_c}{2\omega_0} \left(1 - \frac{\omega_{\pm,k}}{\omega_0} \right) \end{pmatrix}. \quad (\text{D5})$$

The transformation from bare to polaritonic basis in Eq. (D3) can be inverted as

$$\begin{aligned} \hat{a}(\mathbf{k}) &= w_{+,k}^* \hat{p}_+(\mathbf{k}) + w_{-,k}^* \hat{p}_-(\mathbf{k}) - y_{+,k} \hat{p}_+^\dagger(-\mathbf{k}) - y_{-,k} \hat{p}_-^\dagger(-\mathbf{k}), \\ \hat{b}(\mathbf{k}) &= x_{+,k}^* \hat{p}_+(\mathbf{k}) + x_{-,k}^* \hat{p}_-(\mathbf{k}) - z_{+,k} \hat{p}_+^\dagger(-\mathbf{k}) - z_{-,k} \hat{p}_-^\dagger(-\mathbf{k}), \end{aligned} \quad (\text{D6})$$

and the photonic population in the mode \mathbf{k} upon adiabatic switch-off calculated with the Hopfield theory is

thus

$$N'_k = \langle G | \hat{a}^\dagger(\mathbf{k}) \hat{a}(\mathbf{k}) | G \rangle = \sum_{j=\pm} |y_{j,k}|^2. \quad (\text{D7})$$

We can now write Eq. (12) in the limit of vanishing losses, and thus real eigenfrequencies, as

$$\lim_{V(\omega) \rightarrow 0} N_k = \sum_{j=\pm} \left[\frac{\omega_{j,k}^2 - k^2 c^2}{4k^2 c^3} \frac{d\omega_{j,k}}{dk} \right] - \frac{1}{2}. \quad (\text{D8})$$

Plugging Eqs. (D4)-(D5) into Eq. (D7) and Eq. (D8), and developing the heavy but straightforward algebra we can then prove that the two expressions coincide, verifying the correctness of our procedure.

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